Mark Scheme 4725 June 2006

| 1. | i) $\left(\begin{array}{ll}7 & 4 \\ 0 & -1\end{array}\right)$ <br> (ii) $\quad\left(\begin{array}{ll}3 & 0 \\ 0 & 3\end{array}\right)$ $k=3$ | B1 <br> B1 <br> B1 <br> B1 | $\begin{aligned} & 2 \\ & 4 \end{aligned}$ | Two elements correct <br> All four elements correct <br> A-B correctly found <br> Find $k$ |
| :---: | :---: | :---: | :---: | :---: |
| 2 | (i) <br> (ii) $\left(\begin{array}{ll}1 & -1 \\ 0 & 1\end{array}\right)$ | M1 <br> A1 <br> B1 B1 | $\begin{array}{\|l} 2 \\ 4 \end{array}$ | For 2 other correct vertices <br> For completely correct diagram <br> Each column correct |
| 3. | (i) $2+3 \mathrm{i}$ <br> (ii) $p=-4$ $q=13$ | B1 <br> M1 <br> A1 <br> M1 <br> A1 | 1 | Conjugate seen <br> Attempt to sum roots or consider $x$ terms in expansion or substitute $2-3 \mathrm{i}$ into equation and equate imaginary parts <br> Correct answer <br> Attempt at product of roots or consider last term in expansion or consider real parts Correct answer |

\begin{tabular}{|c|c|c|c|c|}
\hline 4. \& \[
\begin{aligned}
\& \Sigma r^{3}+\Sigma r^{2} \\
\& \Sigma r^{2}=\frac{1}{6} n(n+1)(2 n+1) \\
\& \Sigma r^{3}=\frac{1}{4} n^{2}(n+1)^{2} \\
\& \frac{1}{12} n(n+1)(n+2)(3 n+1)
\end{aligned}
\] \& \begin{tabular}{l}
M1 \\
A1 \\
A1 \\
M1 \\
A1
\end{tabular} \& \& \begin{tabular}{l}
Consider the sum as two separate parts \\
Correct formula stated \\
Correct formula stated \\
Attempt to factorise and simplify or expand both expressions \\
Obtain given answer correctly or complete verification
\end{tabular} \\
\hline 5. \& \begin{tabular}{l}
(i) -7 i \\
(ii) \(2+3 \mathrm{i}\)
\[
-5+12 i
\] \\
(iii) \(\frac{1}{5}(4-7 \mathrm{i})\) or equivalent
\end{tabular} \& \[
\begin{array}{|l}
\hline \text { B1 } \\
\text { B1 } \\
\text { B1 } \\
\text { B1 } \\
\text { B1 } \\
\\
\text { M1 } \\
\text { A1 } \\
\text { A1 }
\end{array}
\] \& 2
3
3

3

8 \& | Real part correct |
| :--- |
| Imaginary part correct |
| $\mathrm{i} z$ stated or implied or $\mathrm{i}^{2}=-1$ seen |
| Real part correct |
| Imaginary part correct |
| Multiply by conjugate |
| Real part correct |
| Imaginary part correct |
| N.B. Working must be shown | \\

\hline $6 .$. \& | (i) Circle, Centre $O$ radius 2 |
| :--- |
| One straight line Through $O$ with +ve slope In $1^{\text {st }}$ quadrant only |
| (ii) $1+\sqrt{3}$ | \& | B1 B1 |
| :--- |
| B1 |
| B1 |
| B1 |
| M1 |
| A1 | \& 2 \& | Sketch showing correct features |
| :--- |
| Attempt to find intersections by trig, solving equations or from graph Correct answer stated as complex number | \\

\hline
\end{tabular}

\begin{tabular}{|c|c|c|c|c|}
\hline 7. \& \begin{tabular}{l}
(i)
\[
\mathbf{A}^{2}=\left(\begin{array}{ll}
4 \& 0 \\
0 \& 1
\end{array}\right) \quad \mathbf{A}^{3}=\left(\begin{array}{ll}
8 \& 0 \\
0 \& 1
\end{array}\right)
\] \\
(ii) \(\quad \mathbf{A}^{\mathrm{n}}=\left(\begin{array}{ll}2^{n} \& 0 \\ 0 \& 1\end{array}\right)\) \\
(iii)
\end{tabular} \& \begin{tabular}{l}
M1 \\
A1 \\
A1 \\
B1 \\
B1 \\
M1 \\
A1 \\
A1
\end{tabular} \& 1

4
4

8 \& | Attempt at matrix multiplication |
| :--- |
| Correct $\mathbf{A}^{2}$ |
| Correct $\mathbf{A}^{3}$ |
| Sensible conjecture made |
| State that conjecture is true for $n=1$ or 2 |
| Attempt to multiply $\mathbf{A}^{\mathrm{n}}$ and $\mathbf{A}$ or vice versa |
| Obtain correct matrix |
| Statement of induction conclusion | \\

\hline 8. \& | (i) $\begin{aligned} & a\left[\begin{array}{l} a \\ 0 \\ 2 \end{array} 1\right]-4\left[\begin{array}{ll} 1 & 0 \\ 1 & 1 \end{array}\right]+2\left[\begin{array}{ll} 1 & a \\ 1 & 2 \end{array}\right] \\ & a^{2}-2 a \end{aligned}$ |
| :--- |
| (ii) $a=0 \text { or } a=2$ |
| (iii) (a) |
| (b) | \& | M1 |
| :--- |
| A1 |
| A1 |
| M1 |
| A1A1ft |
| B1 B1 |
| B1 B1 | \& 3

3
3
4
4

10 \& | Correct expansion process shown |
| :--- |
| Obtain correct unsimplified expression |
| Obtain correct answer |
| Solve their $\operatorname{det} \mathbf{M}=0$ |
| Obtain correct answers |
| Solution, as inverse matrix exists or $\mathbf{M}$ nonsingular or $\operatorname{det} \mathbf{M} \neq 0$ |
| Solutions, eqn. 1 is multiple of eqn 3 | \\

\hline
\end{tabular}



\begin{tabular}{|c|c|c|c|c|}
\hline 10 \& \begin{tabular}{l}
\[
\begin{aligned}
\& \text { (i) } \alpha+\beta+\gamma=2 \quad \alpha \beta \gamma=-4 \\
\& \alpha \beta+\beta \gamma+\gamma \alpha=3
\end{aligned}
\] \\
(ii)
\[
\begin{aligned}
\& \alpha+1+\beta+1+\gamma+1=5 \\
\& p=-5
\end{aligned}
\] \\
(iii)
\[
q=-2
\]
\end{tabular} \& \begin{tabular}{l}
B1 B1 \\
B1 \\
M1 \\
A1ft \\
A1ft \\
M1* \\
A1 \\
DM1 \\
A1ft \\
A1ft \\
M2 \\
A1 \\
M1 \\
A2 \\
A1 A1
\end{tabular} \& 3

3

11 \& | Write down correct values |
| :--- |
| Sum new roots |
| Obtain numeric value using their (i) |
| $p$ is negative of their answer |
| Expand three brackets $\alpha \beta \gamma+\alpha \beta+\beta \gamma+\gamma \alpha+\alpha+\beta+\gamma+1$ |
| Use their (i) results |
| Obtain 2 |
| $q$ is negative of their answer |
| Alternative for (ii) $\boldsymbol{\&}$ (iii) |
| Substitute $x=u-1$ in given equation |
| Obtain correct unsimplified equation for $u$ |
| Expand |
| Obtain $u^{3}-5 u^{2}+10 u-2=0$ |
| State correct values of $p$ and $q$. | \\

\hline
\end{tabular}

